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ELECTRICAL AND THERMAL CONDUCTIVITY IN A SUPERDENSE LATTICE. I. HIGH-TEMPERATURE CONDUCTIVITY

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ABSTRACT

The electrical and thermal conductivities are computed analytically for conditions which prevail in white-dwarf cores and the outer layers of neutron stars. We consider here the case in which the temperature is well above the Debye characteristic temperature, and calculate the transition probability for electron-phonon collisions. Because of the peculiar conditions, namely, a Debye length that is much larger than the interatomic spacing, both N- and U-processes can be quite easily included, and a single relaxation time characterizes the interaction. Relativistic effects are taken into account through use of the Dirac wave function for the electron field, and in the relativistic Boltzmann equation. The resulting formula has the exceedingly simple form

$$\sigma = 1.04 \times 10^{22} T_6^{-1} (\rho_6/\mu_e)^{2/3} \text{ sec}^{-1} ,$$

where σ is the electrical conductivity, μ_e is the electron "molecular weight," ρ_6 is the density in units of 10^6 g cm^{-3}, and $T_6 = T/10^6$. The constant is independent of the ion mass, density, and charge. The conductive opacity turns out to be

$$K_c = 2.81 \times 10^{-3} \mu_e^{2/3} \rho_6^{-5/3} T_6^3 .$$

I. INTRODUCTION

The atoms in the outer layers of neutron stars which have cooled to temperatures below 5×10^8 ° K are expected to arrange themselves into a lattice (Abrikosov 1960; Ruderman 1968). In the case of a low-mass star this lattice may fill the entire volume. One expects similar conditions to prevail in white dwarfs (Salpeter 1961). Under these conditions, the atoms of the lattice should be stripped of their electrons, and the electrons form a relativistic degenerate gas. Although Lampe (1968) has shown that electron-electron collisions can be important under nonrelativistic, nondegenerate conditions, the primary source of resistance in this case is electron-phonon interaction, except at low temperatures. This has been established in an order-of-magnitude calculation by Abrikosov (1963). The Debye temperature in normal lattices is replaced in this case by (Mestel and Ruderman 1967)

$$\theta_{\rm D} = \hbar\omega_p/k_{\rm B} \approx 3.9 \times 10^6 \rho_6^{1/2} \text{ °K} , \tag{1}$$

where $\omega_p = (4\pi Z^2 e^2 N_I/M_I \Omega)^{1/2}$ is the plasma frequency (N_I = the number of ions in volume Ω with mass M_I and charge Z; the atomic weight $A = 2Z$ is assumed), $\rho_6 = \rho/10^6$ and $k_{\rm B}$ = Boltzmann's constant. Below $\theta_{\rm D}$, there are competing processes which contribute to the thermal and electrical resistance: phonon-electron, phonon-phonon, and electron-impurity interactions. In a subsequent paper, the low-temperature electron-phonon interaction will be investigated.

Here we calculate the electrical and thermal conductivity in the case $T > \theta_{\rm D}$. *Umklapp*-processes (U-processes) as well as normal processes (N-processes) are taken into account. Screening of the ions is introduced through use of a dielectric constant.

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Relativistic effects are assumed important only in the wave function, which is a solution of the Dirac equation, and in the Boltzmann equation.

II. ELECTRON-PHONON SCATTERING TRANSITION PROBABILITY

The Hamiltonian of the electron-lattice system in equilibrium is[1]

$$H_T = H_I + H_e + H_{eI} , \tag{2}$$

where

$$H_I = \frac{1}{2M_I} \Sigma_\alpha \boldsymbol{P}_\alpha^2 + \Sigma_{\alpha<\beta} V_I(\boldsymbol{R}_\alpha - \boldsymbol{R}_\beta) , \tag{3a}$$

$$H_e = \Sigma_i [\boldsymbol{\alpha}\cdot\boldsymbol{p}_i + \beta m] + \Sigma_{i>j} \frac{e^2}{r_{ij}} , \tag{3b}$$

$$H_{eI} = \Sigma_{\alpha,i} V(\boldsymbol{r}_i - \boldsymbol{R}_\alpha) . \tag{3c}$$

Here H_I describes the interaction of the ions through the potential $V_I(\boldsymbol{R}_\alpha - \boldsymbol{R}_\beta)$; H_e describes the relativistic electrons, and H_{eI} the interaction between electrons and ions. The α_i's and β are the usual Dirac matrices. The following assumptions are made:

1. The total state vector is assumed to be of the form

$$\Psi = \psi_e | l \rangle , \tag{4}$$

where ψ_e is the electronic state and $| l \rangle$ is the lattice state.

2. The lattice Hamiltonian is expressed in standard form by the introduction of creation and annihilation operators (cf. Ziman 1960). The equilibrium positions of the ions are referred to as $\boldsymbol{R}_\alpha{}^0$; the actual positions, $\boldsymbol{R}_\alpha$. For a general motion of the lattice, we express the vibrations as sums over the normal modes:

$$\boldsymbol{R}_\alpha - \boldsymbol{R}_\alpha{}^0 \equiv \boldsymbol{n}_\alpha = \frac{1}{(N_I M_I)^{1/2}} \Sigma_s \boldsymbol{e}_s \xi_s \exp(i\boldsymbol{k}\cdot\boldsymbol{R}_\alpha{}^0) . \tag{5}$$

The sum denoted by the subscript s is over polarization (μ) and wave vectors ($\boldsymbol{k}$) in the first Brillouin zone; $\boldsymbol{e}_s$ is the unit vector; and ξ_s is the wave amplitude of a (μ, $\boldsymbol{k}$)-labeled phonon. The matrix elements of ξ_s between lattice states l and l' are (Ziman 1960)

$$\langle l | \xi_s | l' \rangle = (2\omega_s)^{-1/2} [N_s{}^{1/2} \delta_{N_s, N'_s - 1} + (N_s + 1)^{1/2} \delta_{N_s, N'_s + 1}] , \tag{6}$$

where ω_s is the frequency of an s-labeled phonon and N_s is the number of phonons in this mode.

3. The interaction between electrons is neglected; i.e., the term e^2/r_{ij} is dropped.

4. We have solved the problem of the electrons interacting with the ions in their equilibrium positions to first order by standard perturbation theory. The resulting wave function is given by

$$\psi(x) = \Omega^{-1/2} [1 + \zeta(x, p)] \exp(i\boldsymbol{p}\cdot\boldsymbol{x}) u(p) , \tag{7}$$

where the $u(\boldsymbol{p})$ are the standard spinor solutions of the free-particle Dirac equation and $\zeta(\boldsymbol{x}, \boldsymbol{p})$ is an operator given by

$$\zeta(x, p) = 4\pi Z e^2 N_I \Sigma \boldsymbol{K}_n \frac{\Lambda_+(\boldsymbol{p} + \boldsymbol{K}_n)\ \gamma_0 \exp(-i\boldsymbol{K}_n\cdot\boldsymbol{x})}{\boldsymbol{K}_n{}^2 [E(\boldsymbol{p} + \boldsymbol{K}_n) - E(\boldsymbol{p})]} . \tag{8}$$

Here $\Lambda_+(\boldsymbol{p})$ is the Dirac projection (casimir) operator, the $\boldsymbol{K}_n$'s are reciprocal lattice vectors, $\gamma_0 = \beta$, and $E(p) = (p^2 + m^2)^{1/2}$. Except at a Brillouin-zone boundary, i.e.,

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[1] Natural units ($\hbar = c = 1$) are used throughout.

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where $E(p) = E(p + K_n)$, this is a small correction to the plane-wave solution on the order of the ratio of the Coulomb to the Fermi energy. For relativistic electrons, this can be neglected, and the electronic wave functions taken as Dirac plane waves. (Note that the solution above is for dielectric constant unity. The introduction of a dynamic dielectric constant is considered below. Here, its effect would clearly be to reduce the interactions between electrons and ions, i.e., to make $\zeta(x, p)$ smaller, and the plane wave an even better approximation.)

According to the above, the equilibrium problem is thus assumed solved, and the state vector is

$$\Psi = \psi|l\rangle\,, \tag{9}$$

where ψ is now the free-electron Dirac field.

We now allow the atoms to move from their equilibrium positions. This introduces the perturbation potential

$$V' = \Sigma_a[V(r - R_a) - V(r - R_a^0)] = \Sigma_a \mathbf{n}_a\cdot\nabla V(r - R_a^0)\,. \tag{10}$$

Here r is the electronic position, and V is the screened Coulomb potential:

$$V(r - R_a^0) = \frac{4\pi Ze^2}{(2\pi)^3}\int d^3q\,\frac{\exp[iq\cdot(r - R_a^0)]}{q^2\epsilon(q, 0)}\,. \tag{11}$$

Here we have made two approximations; namely, (1) that the *static* dielectric constant can be used, and (2) that a nonrelativistic (nonretarded) Coulomb potential describes the interaction. Following Canuto (1970), we take

$$q^2\epsilon(q, 0) = q^2 + r_D^{-2}\,, \tag{12}$$

where $\bar{\lambda}_c^2 r_D^{-2} = 2\alpha\mu(\mu^2 - 1)^{1/2}/\pi$ ($\bar{\lambda}_c = m^{-1}$) is the Debye length for a relativistic degenerate electron gas, μ is the Fermi energy in units of m, and $\alpha = e^2$.

The interaction Lagrangian becomes

$$\mathfrak{L}_{\text{int}} = -e\langle l_f|\psi^*{}_f\,\gamma_\mu A_\mu\psi_i|l_i\rangle = -e\langle l_f|\psi^\dagger{}_f\,V'(r - R_a)\psi_i|l_i\rangle \tag{13}$$

and the interaction Hamiltonian is simply

$$H_{\text{int}} = \int d^3r\,\mathfrak{L}_{\text{int}}\,. \tag{14}$$

Thus one has

$$H_{\text{int}} = -\frac{4\pi e^2 Z}{\Omega}\,\frac{q\cdot I}{q^2 + r_D^{-2}}\,u(p')^\dagger u(p)\,, \tag{15}$$

with

$$q = p' - p\,,\quad I = \langle l_f|\Sigma_a \mathbf{n}_a \exp(-iq\cdot R_a)|l_i\rangle\,, \tag{16}$$

and we have introduced the notation p, p' for the initial and final electron momenta; and l_i, l_f for the initial and final lattice states. The matrix element I is evaluated easily with the help of equations (5) and (6) and by using the relation

$$\Sigma_a \exp(il\cdot R_a^0) = N_I\delta(l, K_n)\,, \tag{17}$$

where K_n is a reciprocal lattice vector, and $\delta(l, K_n)$ is the Kronecker delta. Then equation (16) becomes

$$I = (N_I/M_I)^{1/2}\Sigma_s e_s\langle l_f|\xi_s|l_i\rangle\delta(k, q + K_n)\,, \tag{18}$$

where now, for q outside the 1st Brillouin zone, K_n is the reciprocal lattice vector which, added to q, brings it back into this zone (U-processes).

Remembering that

$$k = \begin{cases} q\,, & q \text{ in first Brillouin zone}\,, \\ q + K_n\,, & q \text{ out of first Brillouin zone}\,, \end{cases}$$

and squaring the matrix element, summing over initial and averaging over final spin states, one obtains the transition probability

$$W(p, p') = W_0 \frac{1 + \epsilon\epsilon' + PP' \cos\theta}{\epsilon\epsilon'(Q^2 + \gamma^2)^2} \Big\{ \Sigma_\mu \frac{(Q \cdot e_s)^2}{\epsilon_s} [N_s \delta(\epsilon' - \epsilon - \epsilon_s) + (N_s + 1)\delta(\epsilon' - \epsilon + \epsilon_s)] \Big\}\,, \tag{19}$$

where now $P = p/m$, $Q = q/m$, $\epsilon^2 = P^2 + 1$, $P' \cdot P = PP' \cos\theta$, $\gamma = \lambdabar_c / r_D$, $\epsilon_s = \omega_s/m$, and

$$W_0 \equiv \frac{(2\pi)^3 Z^2 \alpha^2 N_I m}{(\Omega/\lambdabar_c^3)^2 M_I} \frac{1}{\lambdabar_c}\,. \tag{20}$$

Note that the summation is simply over the polarization part of the index s.

III. THE HIGH-TEMPERATURE APPROXIMATION

We must now consider the factors $\Sigma(Q \cdot e_s)^2 N_s/\epsilon_s$ which occur in equation (19). For large T, assuming the phonons are in thermal equilibrium, we have $N_s + 1 \approx N_s = [\exp(\omega_s/k_B T) - 1]^{-1} \approx k_B T/\omega_s$. Thus these factors become

$$\Sigma_\mu \frac{(Q \cdot e_s)^2}{\epsilon_s} N_s \approx m k_B T \Sigma_\mu \frac{(Q \cdot e_s)^2}{\omega_s^2}\,. \tag{21}$$

Now, in the ordinary Debye model, $\omega_s \approx c_s k$, where c_s is the sound speed. However, in the case of the high densities under consideration here, where the Debye screening length is much larger than the interatomic distance, Salpeter (1961) has pointed out that the frequency is practically independent of the wavelength and is simply the plasma frequency. Thus, for all but the smallest k, we can replace ω_s by ω_p, so that

$$\Sigma_\mu \frac{(Q \cdot e_s)^2}{\epsilon_s} N_s \approx \frac{k_B T}{m \omega_p^2 \lambdabar_c^2} Q^2\,. \tag{22}$$

(Actually, a more detailed analysis of the phonon spectrum [Abrikosov 1963] shows that $\omega_s = \omega_p$ only for longitudinal phonons. However, $Q \cdot e_s$ is zero for transverse phonons in N-processes, and can be taken, as an approximation, to equal Q for both N- and U-processes.) Ordinarily for U-processes, where q can be large and k can be small, this would be a bad approximation. But here, the number of cases in which $\omega_s \ll \omega_p$ is so small that the error introduced can be neglected. Thus the U-processes become indistinguishable from the N-processes, and it is a completely straightforward calculation to obtain the relaxation time and finally the thermal conductivity. The relaxation time is well defined as a result of this indistinguishability between the N- and U-processes, since now the transition probability depends only on the magnitude of the momentum transfer, and not its direction.

Finally, we make the approximation that the collisions are elastic, i.e., $\epsilon' - \epsilon \pm \epsilon_s x \epsilon' - \epsilon$. This is a good approximation for all but small scattering angles, since the Fermi momentum is some MeV, while ϵ_s is at most a few keV. This then yields the high-temperature transition probability

$$W(p, p') = 2W_0 \frac{k_B T}{m \omega_p^2 \lambdabar_c^2} \frac{Q^2}{(Q^2 + \gamma^2)^2} \frac{1 + \epsilon\epsilon' + PP' \cos\theta}{\epsilon\epsilon'} \delta(\epsilon' - \epsilon)\,, \tag{23}$$

valid for $T > \theta_D$.

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IV. THE THERMAL AND ELECTRICAL CONDUCTIVITIES

The relativistic Boltzmann equation in three-dimensional notation is given by (Kursunoglu 1961)

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f + (e\boldsymbol{E} + e\boldsymbol{v} \times \boldsymbol{B})\cdot\nabla_p f = \left(\frac{\partial f}{\partial t}\right)_c , \qquad (24)$$

where $\nabla_p = [\partial E(p)/\partial p]\partial/\partial E(p)$ and equation (24) differs from the nonrelativistic equation only in this term. One need only consider equation (24) with $\partial f/\partial t$, ∇f, and $\boldsymbol{B}$ set equal to zero. The resulting equation then corresponds to the application of an electric field to an isothermal medium, and the first-order correction to the distribution function (or density matrix) can then be used to calculate the isothermal electrical conductivity. Since in the high-temperature case under consideration a single unique relaxation time can be calculated, the Widemann-Franz law holds, which relates the thermal and electrical conductivities by the relation (cf. Ziman 1960)

$$K_T = \tfrac{1}{3}\left(\frac{\pi k_{\mathrm{B}}}{e}\right)^2 \sigma T , \qquad (25)$$

where K_T is the thermal and σ is the electrical conductivity coefficient. Thus in this case it suffices to solve the equation

$$e\boldsymbol{E}\cdot\nabla_p f^0(p) = -\Sigma_{p'\neq p} W(p', p)[f^1(p') - f^1(p)] , \qquad (26)$$

where $f^0(p)$ is the density matrix in the unperturbed state, i.e., the Fermi distribution function, and $f^1(p)$ is the first-order correction to the diagonal matrix elements. The right-hand side is the first-order expression for $(\partial f/\partial t)_c$. The equation is solved by standard methods: we set $(\partial f/\partial t)_c = -f^1(p)/\tau(\epsilon)$, and let $f^1(p) = -\chi(p)\ (\partial f^0(p)/\partial\epsilon)$, where $\chi(p) = p_z a(\epsilon)$. Because of energy conservation, the factor $a(\epsilon)$ doesn't enter the formulae.

The relaxation time is then simply

$$\tau^{-1}(\epsilon) = \Sigma\, W(p', p)(1 - \cos\theta) = \frac{\Omega}{(2\pi)^3 \lambdabar_c{}^3} \int d^3P' W(p', p)(1 - \cos\theta), \qquad (27)$$

which becomes, after the energy integration,

$$\tau^{-1}(\epsilon) = \frac{1}{\tau_0}\, \beta^{3/2} \int_{-1}^{1} \frac{dZ}{\epsilon} \frac{(1 - Z)^2(\epsilon^2 + 1 + \beta Z)}{[2\beta(1 - Z) + \gamma^2]^2} \equiv \frac{1}{\tau_0}\, F(\epsilon) ,$$

where we have introduced the notation $Z = \cos\theta$, $\beta = \epsilon^2 - 1$, $\tau_0{}^{-1} = 4\Omega W_0 k_{\mathrm{B}} T/[(2\pi)^2 m\omega_p{}^2 \lambdabar_c{}^5]$. Note that, in contradistinction to a Debye solid, the integration extends over all angles. This is because of the above-mentioned indistinguishability between the N- and U-processes. The integral is simple to evaluate; it turns out to be

$$\epsilon F(\epsilon) = \frac{2\epsilon^2 + 3\gamma^2/4}{8(\epsilon^2 - 1)^{3/2}} [4(\epsilon^2 - 1) + \gamma^2(1 + \delta + 2\ln\delta)] - 2(\epsilon^2 - 1)^{3/2}\delta/\gamma^2 , \qquad (28)$$

where $\delta \equiv \gamma^2/(4\beta + \gamma^2) = \gamma^2/(4\epsilon^2 + \gamma^2 - 4)$. With the relaxation time $\tau(\epsilon)$, the solution to the Boltzmann equation (27) is straightforward:

$$f^1(p) = -e\tau(\epsilon)E_z \frac{\partial f^0(p)}{\partial p_z} . \qquad (29)$$

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Finally, the current is given by

$$J_z = \frac{2e}{(2\pi)^3} \int d^3p \, \frac{\partial E(p)}{\partial p_z} f^1(p) \,, \tag{30a}$$

or, in dimensionless quantities,

$$J_z = \frac{2e}{(2\pi)^3 \bar{\lambda}_c^3} \int d^3P \, \frac{P_z}{\epsilon} f^1(p) \,. \tag{30b}$$

Upon substituting equation (29) into equation (30b) and noting that we can replace P_z^2 by $\frac{1}{3}P^2$ and that $\sigma = J_z/E_z$, we obtain

$$\sigma = -\frac{2e^2\tau^0}{3(2\pi)^3 m \bar{\lambda}_c^3} \int d^3P \, \frac{P^2}{\epsilon^2} [F(\epsilon)]^{-1} \frac{\partial f^0(p)}{\partial \epsilon} \,. \tag{31}$$

Now $d^3P = 4\pi\epsilon(\epsilon^2 - 1)^{1/2}d\epsilon$, and $df^0(p)/d\epsilon \approx -\delta(\epsilon - \mu)$ where μ is the chemical potential in units of m. Thus we have

$$\sigma = \sigma^0 \phi(\mu) \,, \tag{32}$$

where now

$$\sigma^0 = \frac{1}{12\pi^2} \frac{m}{k_B T} \frac{1}{\bar{\lambda}_c} = \frac{5.1843}{T_6} \times 10^{22} \text{ sec}^{-1} \,, \qquad \phi(\mu) = \frac{(\mu^2 - 1)^{3/2}}{\mu F(\mu)} \,, \tag{33}$$

and $F(\mu)$ is given by equation (28). In the fully relativistic degenerate case, when it is noted that $\gamma^2 \approx a\mu^2 \ll \mu^2$, $\phi(\mu)$ becomes to good approximation $2\mu^2 \approx 2(\rho_6/\mu_e)^{2/3}$ and

$$\sigma = 1.03685 T_6^{-1}(\rho_6/\mu_e)^{2/3} \times 10^{22} \text{ sec}^{-1} \,. \tag{34}$$

Here μ_e is the so-called electron molecular weight (cf. Chiu 1968). The remarkable outcome is that *the conditions in the lattice enter only through the temperature and the chemical potential of the electrons!* That is, the conductivity is entirely independent of the mass of the ions, their charge, and their density! This peculiar circumstance arises as a result of the fact that the phonon frequencies are given by the plasma frequency of the ions, for all wavelengths. Equations (19) and (20) plus the discussion following show that the strength of the interaction goes roughly as ω_p^2/ω_s^2, which product contains all the information about conditions in the lattice. When $\omega_s \approx \omega_p$, the interaction is uniformly weak. This is because the amplitude of the phonons goes as ω_s^{-1} (see eq. [6]) which in a Debye solid is usually greater than ω_p^{-1}. Thus we expect, and find, a high value for the conductivity in such a lattice.

The thermal conductive opacity coefficient is related to K_T through the Widemann-Franz law and the relation

$$K_c = \frac{4a}{3\rho} T^3 K_T^{-1} \,, \qquad a = \tfrac{1}{15}\pi^2 k_B^4 \,. \tag{35}$$

Thus we find

$$K_c = \frac{0.3\pi a e^2 \bar{\lambda}_c}{k_B m} \mu_e^{2/3} \rho_6^{-5/3} T^3 \times 10^7 = 2.8095 \mu_e^{2/3} \rho_6^{-5/3} T_6^3 \times 10^{-3} \tag{36}$$

in the ultrarelativistic case. (A more exact formula can be obtained from eqs. [25], [32], [33], and [35].)

The above formulae are applicable for $\rho_6 \gtrsim 10$, and $T_M \gtrsim T > \theta_D$, where T_M is the melting temperature. T_M is generally estimated to lie in the range $75 \leq \Gamma \leq 150$, where $\Gamma = 23\rho_6^{1/3} Z^{5/3} T_6^{-1}$. For $Z > 2$, $T_M > \theta_D$.

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